State Space Adaptive Control for a Rigid Rotor Excited by Non-Conservative Cross-Coupling Forces

P. Wurmsdobler
Institute for Machine and Process Automation
University of Technology Vienna, Austria

Abstract

In this paper an adaptive state space controller for a rigid rotor suspended in active magnetic bearings is presented. Based on a discrete time state space model in controller canonical form a recursive adaptation algorithm is used to estimate both all system parameters and all states. A pole placement controller with additional integrative feedback is then calculated upon the identified model. The entire control concept was tested by means of simulation. Simulation runs of the closed loop system including the proposed algorithm show the successful operation for changes of system parameters, in this case the sudden appearance of non-conservative cross-coupling forces, as generated by seals, for example.

1 Introduction

In conventional rotor-bearing systems with journal bearings and seals, non-conservative cross-coupling forces may lead to instability. A possible way to cope with this problem is to reduce the influence of the cross-coupling mechanism by applying higher damping to the system, which means dissipation of energy.

If a rotor is levitated by magnetic bearings, one can make use of the controllability of the entire system by active magnetic bearings. In many active magnetic bearing applications the design of the entire control system is performed under the assumption that all system parameters are exactly known in advance and do not change during operation. For these type of plants adaptive control is not necessary. It is sufficient to design a robust controller for a limited range of operation.

In high performance turbomachinery, however, a change in system parameters, such as a sudden change of pressure in a turbo pump may easily occur. This event may lead to a change in stiffness parameters of a sealing and finally to destabilising non-conservative cross-coupling forces, for example. In this case an adaptive controller is absolutely necessary. Non-conservative forces can then be compensated by control forces without loss of stability and without applying more damping to the rotor-bearing system by means of on-line identification and adaptive control techniques.

2 System modelling

In order to apply state space adaptive control to a system, a comprehensive model has to be established for two reasons. Firstly, a precise model is needed for simulation purposes, and secondly a linear state space model can be derived from the previous one. In the following all subsystems used for the simulation model and for the derivation of a linear state space model are presented (see [Wurmsdobler, 1997] for a detailed description).

Digital controller: The controller executes the control laws within the sampling period $T_s = 100\,\mu$s. The quantisation, however, is symbolically ascribed to the ADCs and DACs.

DAC for control voltage: The DAC (Digital to Analogue Converter) is modelled by a series of saturation, quantiser and time delay.

PWM with switching amplifier: The PWM (Pulse Width Modulator) in conjunction with the switching amplifier is modelled as a saw-tooth generator modulating the input, a comparator, and two voltage sources.

Bearing actuator model: All magnetic actuators are equal in design and consist of four electromagnets. For each electromagnet the well known equations for an electro magnetic circuit are used with the switched voltage as input and the coil current and the magnetic force as output.

Rotor model: The second order equation of motion for a rigid rotor with its degrees of freedom expressed by the bearing coordinates are used with the actuator forces as input and the rotor positions as output.

Position sensors and ADC for measured rotor positions: The measurement process is short cut by the conversion constant k_m. The ADC (Analogue to Digital Converter) is simulated by a series of quantiser, time delay and saturation.

ADC for measured currents: The conversion process is simulated by a series of quantiser, time delay and saturation.

2.1 Model linearisation

The goal for the linearisation process is to derive a linear state space model of the nonlinear rotor bearing system as described above for the point of operation defined by steady state values for all state variables involved in the rotor bearing system. In the present investigation these are the rotor position, the rotor velocity, and all fluxes of all electromagnets of both magnetic actuators defined by the bias current. This bias current is $i_0 = 4\,\mathrm{A}$ for the present application. The rotor position and velocity are zero for the point of operation.

In order to make a linearisation at the point of operation possible, a current control loop is introduced for all electromagnets. This procedure reduces the order of the state space model as long as the bandwidth of the current control loop covers the frequency band of the position control loop (see [Wurmsdobler, 1997] for more details).

2.1.1 Linearised actuator model

The linearised model of the active magnetic bearings is given by the the current gain factor K_i and the position stiffness K_s as

$$K_i = \frac{4\,N^2\,i_0}{\mathcal{R}_0^2\,\mu_0\,A}\cos\frac{\alpha}{2}, \qquad \text{and} \qquad K_s = -\frac{8\,N^2\,i_0^2}{\mathcal{R}_0^3\,\mu_0^2\,A_l^2}\cos\frac{\alpha}{2}, \tag{1}$$

with the number of windings N, the initial reluctance $\mathcal{R}_0$, the magnetic permeability in vacuum μ_0, the cross section area A, and the angle between the pole shoes α.

2.1.2 Linear continuous time state space model for the rotor bearing system

The position control loop basically consists of the magnetic actuator, the rotor, the sensors, the ADCs and the controller. The conversion delay of $3\,\mu$s for the ADCs is negligible in comparison with the sample time of $100\,\mu$s. The conversion gain from m to μm is ascribed to the rotor model, which is linear itself. Neglecting the transfer characteristics of the sensors, ADCs and DACs, the resulting continuous time state space model is then given in bearing coordinates in the form

$$\dot{\boldsymbol{x}}_b = \boldsymbol{A}\cdot\boldsymbol{x}_b + \boldsymbol{B}\cdot\boldsymbol{u}, \tag{2}$$

$$\boldsymbol{y} = \boldsymbol{C}\cdot\boldsymbol{x}_b, \tag{3}$$

with the continuous time state vector $\boldsymbol{x}_b = [\boldsymbol{z}_b, \dot{\boldsymbol{z}}_b]^T$, the output vector $\boldsymbol{y} = k_m\boldsymbol{z}_b$, and the input vector $\boldsymbol{u}$ and the rotor positions in bearing coordinates $\boldsymbol{z}_b$.

Although the position is measured in the sensor planes, the proximity sensors are assumed to be collocated with the bearings for the linear model. This is justified for a rigid rotor model, because the displacements in the bearing planes $\boldsymbol{z}_b$ can easily be calculated from the sampled sensor signal by the simple transformation. The system matrices are defined as follows:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{M}_b^{-1}(\boldsymbol{N}_b + \boldsymbol{K}_s) & -\boldsymbol{M}_b^{-1}\boldsymbol{G}_b \end{bmatrix}, \tag{4}$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{M}_b^{-1}\boldsymbol{K}_i \end{bmatrix}, \tag{5}$$

$$\boldsymbol{C} = \begin{bmatrix} k_m\mathbf{I} & \mathbf{0} \end{bmatrix}, \tag{6}$$

with the mass matrix $\boldsymbol{M}_b$, the gyroscopic matrix $\boldsymbol{G}_b$, the position stiffness matrix $\boldsymbol{K}_s$, the current gain matrix $\boldsymbol{K}_i$, and the matrix of the non-conservative cross coupling forces $\boldsymbol{N}$ expressed in bearing coordinates.

2.1.3 Deterministic discrete time state space model for the rotor bearing system

A transformation into a discrete time system with sampling time T_s yields a deterministic discrete state space model of no particular structure. Since the discrete state space model should have a minimum number of parameters to be identified, it is transformed into a canonical form. Numerical errors accompanied by that form can be avoided by scaling the measured input to μm instead of meters. With respect to the calculation of an adaptive controller, a controller canonical form is chosen [Tolle, 1985]. The entire deterministic model in canonical form introducing 64 parameters is then

$$\mathbf{x}(k+1) = \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,\mathbf{u}(k), \tag{7}$$

$$\mathbf{y}(k) = \mathbf{C}\,\mathbf{x}(k), \tag{8}$$

with the (8×1)-state vector $\mathbf{x}(k)$. The (4×1)-input vector $\mathbf{u}(k)$ is the sampled input vector $\boldsymbol{u}(t)$ and the (4×1)-output vector $\mathbf{y}(k)$ is the sampled vector $\boldsymbol{y}(t)$. The (8×8)-system matrix $\mathbf{A} = \{\mathbf{A}^{(ij)}\}$ is partitioned into the ν_i-dimensional sub-matrices of the form

$$\mathbf{A}^{(ii)} = \begin{bmatrix} 0 & 1 \\ -a_2^{(ii)} & -a_1^{(ii)} \end{bmatrix}, \qquad \text{and} \qquad \mathbf{A}^{(ij)} = \begin{bmatrix} 0 & 0 \\ -a_2^{(ij)} & -a_1^{(ij)} \end{bmatrix}, \tag{9}$$

with $i, j = 1, 2, 3, 4$ and ν_i being the structural indices [Tolle, 1985]. The (8×4)-control matrix $\mathbf{B} = \{\boldsymbol{b}^{(ij)}\}$ is composed of

$$\boldsymbol{b}^{(ii)} = \begin{bmatrix} 0 & 1 \end{bmatrix}^T, \qquad \text{and} \qquad \boldsymbol{b}^{(ij)} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \tag{10}$$

and the(4×8) measurement matrix $\mathbf{C} = \{\boldsymbol{c}^{T(ij)}\}$ is

$$\boldsymbol{c}^{T(ij)} = \begin{bmatrix} c_2^{(ij)} & c_1^{(ij)} \end{bmatrix}. \tag{11}$$

3 State space adaptive control

A regulator is a feedback mechanism by which a system is guided along a desired behaviour. The design of this regulator is commonly based upon a mathematical model of the system to be controlled. Such a model as derived in the previous section is treated as a true description of the system and is expected not to change in terms of its structure. The plant parameters, however, can be slowly time-variant or may be uncertain from the beginning. In these cases, adaptive control is necessary. In the following a model identification adaptive control algorithm is presented.

3.1 State and parameter estimation

Based upon the deterministic state space model defined by Eqn.(7) and Eqn.(8), a predictor for the output $\mathbf{y}(k)$ including the Kalman matrix $\mathbf{K}$ is introduced as

$$\hat{\mathbf{x}}(k+1, \mathbf{p}) = \mathbf{A}(\mathbf{p})\,\hat{\mathbf{x}}(k, \mathbf{p}) + \mathbf{B}\,\mathbf{u}(k) + \mathbf{K}(\mathbf{p})\,\varepsilon(k), \tag{12}$$

$$\mathbf{y}(k) = \mathbf{C}(\mathbf{p})\,\hat{\mathbf{x}}(k, \mathbf{p}) + \varepsilon(k), \tag{13}$$

where $\varepsilon(k)$ corresponds to the prediction error or *innovation* with

$$\varepsilon(k, \mathbf{p}) = \mathbf{y}(k) - \hat{\mathbf{y}}(k|\mathbf{p}), \tag{14}$$

and $\hat{\mathbf{y}}(k|\mathbf{p})$ is the estimated output of the system under the assumption of the parameter vector $\mathbf{p}$. The Kalman matrix is parameterised as $\mathbf{K}(\mathbf{p}) = \{\mathbf{k}^{(ij)}\}$ with $i, j = 1, 2, 3, 4$ and

$$\mathbf{k}^{(ij)} = \left[\begin{array}{cc} k_2^{(ij)} & k_1^{(ij)} \end{array} \right]^T . \tag{15}$$

All parameters of this innovations model defined by the matrices $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{K}$ are summarised within the n_p-dimensional parameter vector

$$\mathbf{p} = \left[-a_2^{(11)}, -a_1^{(11)}, \ldots, c_2^{(11)}, c_1^{(11)}, \ldots, k_2^{(11)}, k_1^{(11)}, \ldots \right] . \tag{16}$$

The Kalman matrix introduces 32 more parameters to the entire estimation algorithm, which means that there are $n_p = 96$ parameters to be estimated. Based upon the innovations model a recursive algorithm has to be implemented to estimate the system states, all system parameters and the Kalman matrix under on-line conditions for a controlled system with stochastic disturbances. It must be admitted that the number of parameters to be estimated is rather high, but the numerical implementation of the adaptation algorithm should be possible due to the simplicity of the numerical operations involved.

3.1.1 The recursive prediction error method

The *recursive prediction error method* (RPEM, [Ljung and Søderstrøm, 1983]) applied to the innovations model is used in this work. This algorithm can estimate both system parameters and the system states. An innovations model is used to predict the system states $\hat{\mathbf{x}}(k)$ and the output $\hat{\mathbf{y}}(k)$ based upon the system inputs $\mathbf{u}(k)$ and its parameters $\hat{\mathbf{p}}(k)$. The difference between model output $\hat{\mathbf{y}}(k)$ and the measured system output $\mathbf{y}(k)$, the prediction error $\boldsymbol{\varepsilon}(k)$, is then used for updating the system parameters $\hat{\mathbf{p}}(k)$. These parameters are later on used for the controller design. The implemented algorithm can be summarised by the following equations with the estimated parameter vector $\hat{\mathbf{p}}$. These equations have to be evaluated at each sample time interval.

1. Compute the estimation error $\boldsymbol{\varepsilon}$ from the measured output and the previous estimate such that

$$\boldsymbol{\varepsilon}(k) = \mathbf{y} - \hat{\mathbf{y}}(k|\hat{\mathbf{p}}(k-1)). \tag{17}$$

2. Compute the auxiliary matrix

$$\mathbf{L}(k) = \mathbf{P}(k-1)\, \boldsymbol{\Psi}(k) \left(\mathbf{I} + \boldsymbol{\Psi}^T(k)\, \mathbf{P}(k-1)\, \boldsymbol{\Psi}(k) \right)^{-1} . \tag{18}$$

3. Then compute the new parameter estimate $\hat{\mathbf{p}}$ such that

$$\hat{\mathbf{p}}(k) = \hat{\mathbf{p}}(k-1) + \mathbf{L}(k)\, \boldsymbol{\varepsilon}(k), \tag{19}$$

4. and update the covariance matrix

$$\mathbf{P}(k) = \frac{1}{\rho(k)} \left(\mathbf{P}(k-1) - \mathbf{L}(k)\, \boldsymbol{\Psi}^T(k) \mathbf{P}(k-1) \right) . \tag{20}$$

5. For the use with the state space controller compute the estimated states

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}(\hat{\mathbf{p}}(k))\,\hat{\mathbf{x}}(k) + \mathbf{B}\,\mathbf{u}(k) + \mathbf{K}(\hat{\mathbf{p}}(k))\,\boldsymbol{\varepsilon}(k), \tag{21}$$

6. and the estimated outputs $\hat{\mathbf{y}}$ for the next cycle

$$\hat{\mathbf{y}}(k+1) = \mathbf{C}(\hat{\mathbf{p}}(k))\,\hat{\mathbf{x}}(k+1). \tag{22}$$

7. The computation of the auxiliary matrix

$$\begin{aligned}\mathbf{W}(k+1,\hat{\mathbf{p}}(k)) &= \Big(\mathbf{A}(\hat{\mathbf{p}}(k)) - \mathbf{K}(\hat{\mathbf{p}}(k))\mathbf{C}(\hat{\mathbf{p}}(k))\Big)\,\mathbf{W}(k,\hat{\mathbf{p}}(k-1)) \\ &\quad + \mathbf{M}_k - \mathbf{K}(\hat{\mathbf{p}}(k))\,\mathbf{V}_k,\end{aligned} \tag{23}$$

8. yields the gradient matrix

$$\boldsymbol{\Psi}(k+1) = \mathbf{W}^T(k+1,\hat{\mathbf{p}})\,\mathbf{C}^T(\hat{\mathbf{p}}(k)) + \mathbf{V}_k^T(k+1). \tag{24}$$

The matrices $\mathbf{M}_k = \mathbf{M}_k(\mathbf{p},\hat{\mathbf{x}}(k),\boldsymbol{\varepsilon}(k))$ and $\mathbf{V}_k = \mathbf{V}_k(\mathbf{p},\hat{\mathbf{x}}(k))$ are model specific and have to be calculated only once in terms of their structure [Wurmsdobler, 1997].

3.1.2 Forgetting factor

Special attention has to be paid to the so-called forgetting factor $\rho(k)$ in Eqn.(20). This factor determines the rate of change for the covariance matrix and can be compared to a gain for an ordinary control loop. If this factor is too small ($\rho(k) \ll 1$), the covariance matrix increases too fast, if it is too large (close to 1), the algorithm reacts too slowly to parameter changes. Therefore, the forgetting factor needs to be controlled separately by a statistical value, which gives the change of the variance of the estimation error. This value

$$\delta(k) = \frac{\hat{\sigma}_\varepsilon^2(N_1,k) - \hat{\sigma}_\varepsilon^2(N_2,k)}{\hat{\sigma}_\varepsilon^2(N_2,k)}, \tag{25}$$

is introduced with $N_1 \leq N_2$ and $\hat{\sigma}_\varepsilon^2(N_i,k)$ as estimate for the prediction error variance with a variable memory N_i. The idea behind this algorithm is, that under the assumption of a stationary process, the variance of the estimation error in a steady state should be stationary as well. If $\delta(k)$ triggers the threshold δ_H, e.g. 20% of the mean variance, then the parameters of the controlled system are assumed to have changed which causes higher estimation error. The consequences are:

1. Reset the forgetting factor $\rho(k)$ to a smaller value ρ_0,
2. Add a white noise signal of certain deflection to the set point.

The first action causes the increase of the covariance matrix on the one hand, and weight the estimated values in its memory less than the innovation on the other hand. With the second action the closed loop system is excited by an uncorrelated signal. If $\delta(k)$ falls beyond the threshold δ_H again, $\rho(k)$ follows a law

$$\rho(k) = k_\rho\,\rho(k-1) + (1-k_\rho)\,\rho_\infty, \tag{26}$$

with ρ_∞ the final value and k_ρ determining the rate of change. Then, assuming that the new parameters are already found, the covariance decreases to a minimum value.

3.1.3 Stability of the estimation algorithm

The most interesting problem about the estimation algorithm is its stability. As proved in [Ljung and Søderstrøm, 1983], the stability of the estimation algorithm is given, if

$$\left|\mathrm{eig}\left(\mathbf{A}(\mathbf{p}) - \mathbf{K}(\mathbf{p})\,\mathbf{C}(\mathbf{p})\right)\right| < 1, \tag{27}$$

i.e. if the predictor based upon the innovations model defined by Eqn.(12) is stable. Therefore, the eigenvalues of the predictor have to be calculated each time step, when a new estimate has been calculated. An observation is not used if it causes the predictor to be unstable and the parameter vector is only updated if the resulting predictor is stable.

3.2 Controller design

Once the parameters of the system are estimated, the state space controller can be computed based upon the identified model with the assumption that the system parameters are estimated properly (*certainty equivalence principle*, [Goodwin and Sin, 1984]).

In this work a pole placement controller is chosen, because stability has to be guaranteed from the beginning based upon a precalculated model. Another reason for pole placement is that active magnetic bearing systems are viewed as mechanical systems with eigenfrequencies and a certain bandwidth. By means of pole placement specific dynamic characteristics can be applied to the controlled system.

The control law for a state space controller with integrative feedback carried out by the integration of the control variable is used in the form

$$\begin{aligned} \mathbf{u}(k) &= \mathbf{u}_I(k) - \mathbf{K}_x\,\hat{\mathbf{x}}(k), && (28)\\ \mathbf{u}_I(k+1) &= \mathbf{u}_I(k) + \mathbf{K}_I\,\mathbf{e}(k), && (29)\\ \mathbf{e}(k) &= \mathbf{w}(k) - \mathbf{y}(k). && (30) \end{aligned}$$

with the integrative feedback gain matrix $\mathbf{K}_I = \{k_I^{(ij)}\}$ and $i, j = 1, 2, 3, 4$. The controller gain $\mathbf{K}_x = \{\mathbf{k}_x^{T\,(ij)}\}$ is parameterised by

$$\mathbf{k}_x^{T\,(ij)} = \left[\begin{array}{cc} k_{x_2}^{(ij)} & k_{x_1}^{(ij)} \end{array}\right]. \tag{31}$$

Applying the z-transformation for Eqn.(28), Eqn.(7) and Eqn.(8), the transformed output of the controlled system with simple state feedback results in

$$\mathbf{Y}(z) = \mathbf{C}\left(z\,\mathbf{I} - \mathbf{A} + \mathbf{B}\,\mathbf{K}_x\right)^{-1}\mathbf{B}\,\mathbf{U}_I(z). \tag{32}$$

With the convention making use of the controller canonical form [Nazaruddin, 1994]

$$\mathbf{N}\,\mathbf{D}^{-1} = \mathbf{C}\left(z\,\mathbf{I} - \mathbf{A} + \mathbf{B}\,\mathbf{K}_x\right)^{-1}\mathbf{B}, \tag{33}$$

the system output can be rewritten as

$$\mathbf{Y}(z) = \mathbf{N}\,\mathbf{D}_I^{-1}\,\mathbf{K}_I\,\mathbf{W}(z), \tag{34}$$

with the closed loop polynomial matrix

$$\mathbf{D}_I = (z-1)\,\mathbf{D} + \mathbf{K}_I\,\mathbf{N}. \tag{35}$$

The essence of the controller design is that the matrix $\mathbf{D}_I$ is set equal to a desired polynomial matrix $\mathbf{P}_I$, which is a diagonal matrix with the elements $P_I^{(ii)}(z)$ of order $\nu_i + 1$ such as $\mathbf{P}_I = \operatorname{diag} P_I^{(ii)}(z)$ with $i = 1, 2, 3, 4$ with the desired polynomials for the closed loop system with integrative feedback

$$P_I^{(ii)}(z) = z^3 + p_{I_1}^{(ii)} z^2 + p_{I_2}^{(ii)} z + p_{I_3}^{(ii)}. \tag{36}$$

The integrative feedback gain can be calculated from the steady state condition as

$$\mathbf{K}_I = \mathbf{P}_I(z)\,\mathbf{N}^{-1}(z)\Big|_{z=1}. \tag{37}$$

What remains is the computation of the polynomials in $\mathbf{D} = \operatorname{diag} P^{(ii)}(z)$, with the desired polynomials $P^{(ii)}(z)$ of the closed loop system without integrative feedback,

$$P^{(ii)}(z) = z^2 + p_1^{(ii)} z + p_2^{(ii)}. \tag{38}$$

Heuristically speaking, this means how to chose the polynomials $P^{(ii)}(z)$ in order to achieve the closed loop polynomials $P_I^{(ii)}(z)$ based upon the integrative feed back gain $\mathbf{K}_I$. Without claim on generality, the following solution is proposed

$$p_1^{ii} = p_{I_1}^{ii} + 1, \tag{39}$$

$$p_2^{ii} = \sum_{j=1}^{4} c_2^{ji}\, k_w^{(ij)} - p_{I_3}^{ii}. \tag{40}$$

The poles of the closed loop system without integrative feedback are determined by the closed loop system matrix $\mathbf{A}_{cl} = \mathbf{A} - \mathbf{B}\mathbf{K}_x$ and the system matrices $\mathbf{A}$ and $\mathbf{B}$ being in controller canonical form. The coefficients of the control matrix can be easily calculated in the form

$$k_{x_m}^{(ij)} = a_m^{(ij)}, \tag{41}$$

$$k_{x_m}^{(ii)} = a_m^{(ii)} + p_m^{(ii)}, \tag{42}$$

with $m = 1, 2$ and $i, j = 1, 2, 3, 4$ [Nazaruddin, 1994].

Here, an advantage of a model description in controller canonical form becomes obvious. All parameters of the state space controller can be calculated directly from the state space model using simple algebra.

4 Simulation results

In order to simulate a change in system parameters, the sudden appearance of non-conservative cross-coupling forces as generated by seals of rotating machinery rotors is assumed. These forces change the system matrix by changing the skew-symmetric cross-coupling terms of the stiffness matrix in a given rotor plane. The step is considered to be the worst case in turbomachinery application, e.g. a sudden pressure loss or leakage in sealings leads to a parameter change in a linear model. Actually, the effect of the cross-coupling forces develops slowly, and an abrupt change in the non-conservative stiffness parameter can be considered as a worst case scenario.

4.1 Initial values

For the simulation of a parameter change all initial state values are set equal to the initial values for a set point change as described in the previous section, namely $\mathbf{x}(t = 0) = \mathbf{0}$, $\hat{\mathbf{x}}(k = 0) = \mathbf{0}$, $\mathbf{u}(k = 0) = \mathbf{0}$, and $\boldsymbol{\varepsilon}(k = 0) = \mathbf{0}$.

The initial parameters for the estimation algorithm are set equal to the *a priori* values, derived from the linear state space model in controller canonical form. The initial covariance is set to a rather small value of $\mathbf{P} = 10^{-8} \cdot \mathbf{I}$. The covariance is only increased if a parameter change is triggered by using the forgetting factor.

The forgetting factor $\rho(k)$ determines the change of the covariance matrix, i.e. its increase or decrease. The initial value is chosen to be $\rho(k = 0) = 1$ which corresponds to practically no forgetting, or heuristically speaking, the initial parameters are believed to be the true ones.

4.2 System response due to parameter change

The controlled rotor bearing system is then exposed to a parameter change at $t = 0.01\,\mathrm{s}$ applied to the system by a sudden appearance of cross-coupling forces with a skew-symmetric non-conservative stiffness factor of $k_n = 6 \cdot 10^6$ N/m at a given plane along the rotor axis.

Immediately after the parameter change the system is unstable in terms of its structure, but the displacements increase slowly. When a certain signal to noise ratio is achieved, the parameter change is recognised by the algorithm. In other words, if the variance of the prediction error increases and thus the forgetting control factor $\delta(k)$ presented in Eqn.(25) passes a certain threshold, the parameters are assumed to have changed. Therefore, the forgetting factor is set to a smaller value, in this case $\rho_0 = 0.999$. The corresponding plots can be seen in Fig. 1.

In addition to the reset of $\rho(k)$, a white noise signal with the arbitrarily chosen maximum deflection of $100\,\mu$m is added to the set point. Since this part of the set point error is not correlated to the estimation error generated by the measurement noise, the signal to noise ration increases which makes the algorithm converge faster. The burst signal is added to the set point as long as $\rho(k)$ remains reset, i.e. as long as $\delta(k)$ is larger than the threshold.

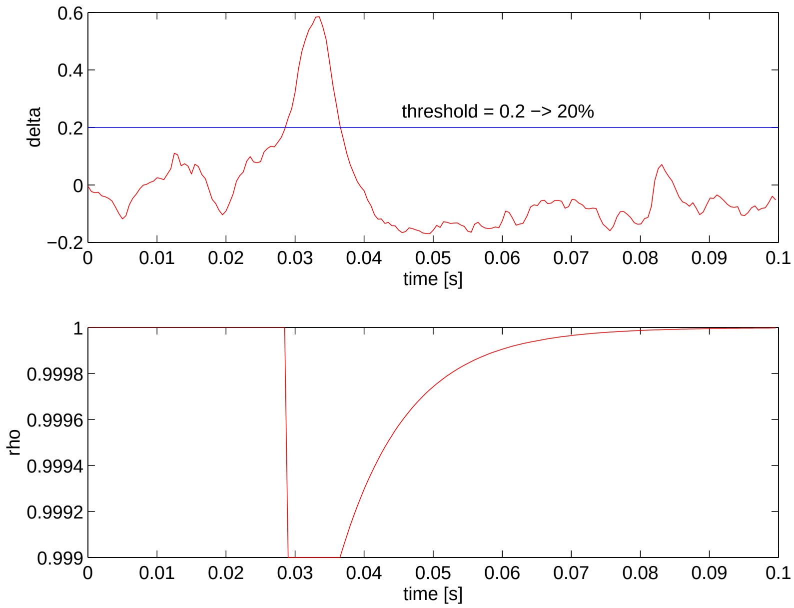

Figure 1: Time history of the forgetting control variable δ and the forgetting factor ρ, after the appearance of a non-conservative cross-coupling stiffness of $k_n = 6 \cdot 10^6\,\mathrm{N/m}$ at time $t = 0.01\,\mathrm{s}$.

Parallel to the identification, the controller parameters are calculated according to Eqn.(41) and Eqn.(42). It can be observed that the feedback gain matrix of the state space controller becomes non-symmetric to the same extent as the parameter of the non-conservative stiffness does. This effect can easily be seen in Eqn.(41). These non-symmetric entries in the controller matrix cause a force, which directly compensates the non-conservative forces of the system. Due to the adaptation of the controller the rotor bearing system can recover from a destabilising parameter change. The time history of the rotor displacements and the control currents can be seen in Fig. 2 and Fig. 3, respectively.

In total $n_p = 96$ parameters are adapted, but only few of them change significantly. This is due to the fact that not every parameter is affected by the change of the non-conservative stiffness factor to the same extent. Note that the parameters in the parameter vector $\mathbf{p}$ depend on discrete time state space matrices, and have neither a specific unit nor a unique physical meaning.

The parameter $\mathbf{p}(1)$, is equal to $-a_2^{11}$ of the system matrix $\mathbf{A}$ and depends mainly on the rotor mass. Therefore, it is obvious that this parameter does not change significantly. The time history of this parameter can be seen in Fig. 5 in Fig. 4.

A second parameter, $\mathbf{p}(8)$ is equal to to $-a_1^{14}$, a non-symmetric entry of the system matrix $\mathbf{A}$ which is of course affected by the skew-symmetric non-conservative stiffness parameter. The time history of this parameter can be seen in Fig. 5.

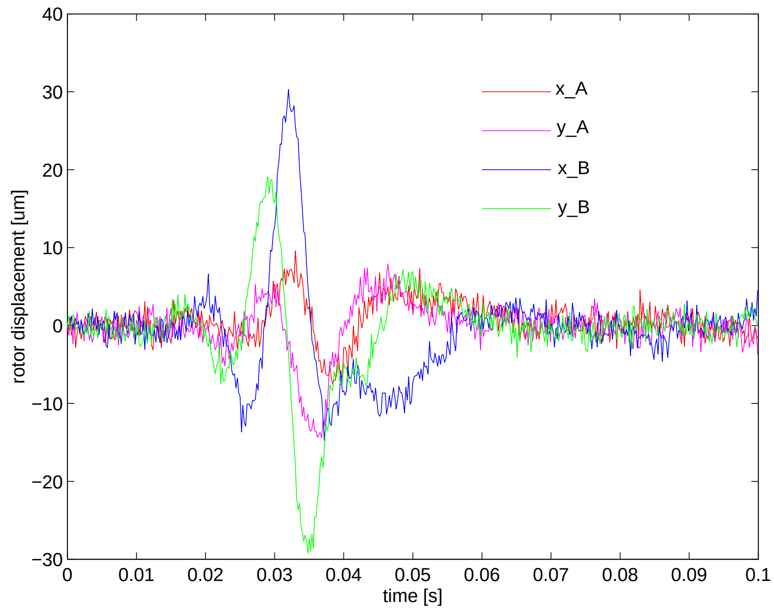

Figure 2: Time history of the rotor displacements after the appearance of a non-conservative cross-coupling stiffness of $k_n = 6 \cdot 10^6\,\mathrm{N/m}$ *at time* $t = 0.01\,\mathrm{s}$.

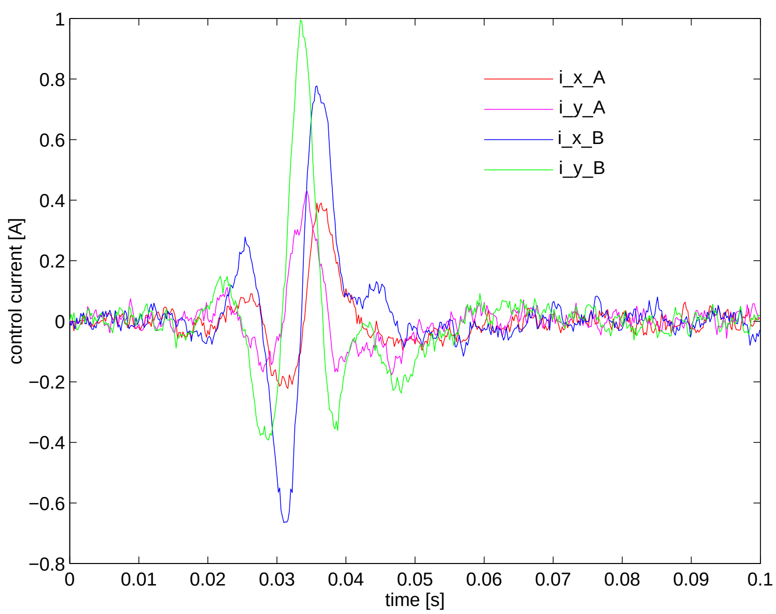

Figure 3: Time history of the control currents after the appearance of a non-conservative cross-coupling stiffness of $k_n = 6 \cdot 10^6\,\mathrm{N/m}$ *at time* $t = 0.01\,\mathrm{s}$.

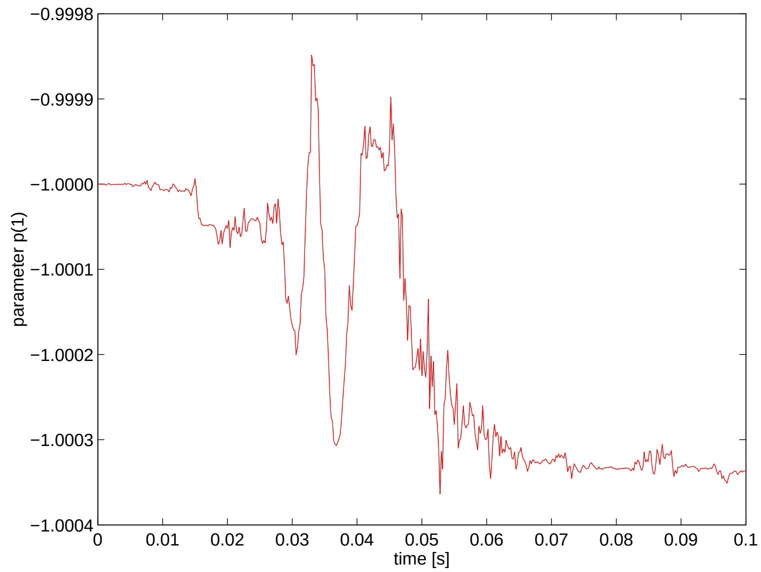

Figure 4: Time history of a sample system parameter $\mathbf{p}(1)$ *after the appearance of a non-conservative cross-coupling stiffness of* $k_n = 6 \cdot 10^6\,\mathrm{N/m}$ *at time* $t = 0.01\,\mathrm{s}$.

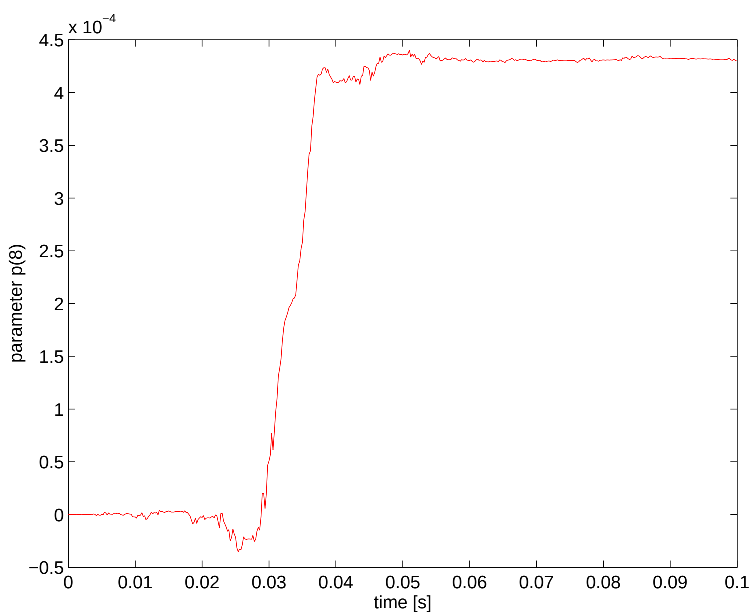

Figure 5: Time history of a sample system parameter $\mathbf{p}(8)$ *after the appearance of a non-conservative cross-coupling stiffness of* $k_n = 6 \cdot 10^6\,\mathrm{N/m}$ *at time* $t = 0.01\,\mathrm{s}$.

5 Conclusions

If a parameter change destabilises the system, the closed loop poles are moved outside the unit-circle in the z-plane, or into the right half complex plane in the continuous time domain. The new poles determine the speed of the destabilisation process, i.e. the time delay until the rotor touches the backup bearing or the amplifiers reaches saturation. This means that the adaptation algorithm has to adapt the system parameters and hence the controller parameters within a certain time and without making the controller run into saturation. Roughly speaking, the adaptation law has to move the closed loop poles back into the stability region, i.e. inside the unit circle.

The speed of the adaptation algorithm is determined by the value of the forgetting factor and the covariance of the parameters to be estimated. Of course, a large covariance covers many parameter changes for systems with slow dynamics. As for the presented system, a large covariance destabilises the system, because unstable predictors are generated permanently, which violate the stability criterion for the adaptation procedure. Therefore, the covariance has to be kept rather small, which could be proved by means of simulation as well. Otherwise the entire system becomes unstable even if there is no parameter change.

The forgetting factor is the second parameter determining the adaptation speed in two ways. On the one hand, a low forgetting factor means that the parameters are considered to be false and the newly estimated are "more true". Then the parameters can change quickly and adapt to the new situation, but generate unstable predictors as well, which destabilise the adaptive control loop in the end. On the other hand the covariance is increased fast if the forgetting factor is too low. This leads to the aforementioned problem of unstable predictors. What remains is a dilemma. Either the system is destabilised by parameter changes with the adaptation algorithm being too slow due to a small covariance, or by unstable predictors with the covariance being too large.

Whether the adaptation algorithm can follow a parameter change finally depends on the covariance and the forgetting factor within certain constraints:

- If the initial covariance is too large from the beginning ($\mathbf{P} > 10^{-6} \cdot \mathbf{I}$), unstable predictors result.
- If the covariance is small enough ($\mathbf{P} < 10^{-6} \cdot \mathbf{I}$), and if
  - the forgetting factor is too small ($\rho(k) \ll 0.999$) a large variance of the system parameters is caused which finally destabilises the system.
  - the forgetting factor is chosen properly ($\rho(k) \approx 0.999$), the adaptation algorithm can cope with the parameter change.
  - the forgetting factor is too large ($\rho(k) \approx 1$), the system runs into saturation before the algorithm adapts to the true parameters.

As a conclusion it can be pointed out that the algorithm for adaptive control presented in this work performs excellently for a certain range of parameter changes. If these are too large instability cannot be prevented.

6 Acknowledgements

The author expresses his honest thanks to the Austrian Science Foundation ("Fonds zur Förderung der wissenschaftlichen Forschung", FWF) for supporting this project.

References

[Bierman, 1988] Bierman, G. (1988). *Factorisation Methods for Discrete Sequential Estimation.* Academic Press, New York.

[Goodwin and Sin, 1984] Goodwin, C. and Sin, K. (1984). *Adaptive Filtering Prediction and Control.* Information and System Science Series, edited by Kailath, T. Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632.

[Ljung and Søderstrøm, 1983] Ljung, L. and Søderstrøm, T. (1983). *Theory and Practice of Recursive Identification.* MIT Press, Cambridge Massachusetts.

[Lang, 1997] Lang, O. (1997). *Vibration Control of a Self-Excited Rotor by Active Magnetic Bearings.* Dissertation, Institut für Maschinendynamik und Meßtechnik, Technische Universität Wien.

[Nazaruddin, 1994] Nazaruddin, Y. (1994). *Adaptive Reglung von Ein- und Mehrgrößensystemen auf der Basis der Zustandsraumdarstellung.* Dissertation, Ruhr Universität Bochum.

[Schweitzer et al., 1993] Schweitzer, G., Traxler, A., and Bleuler, H. (1993). *Magnetlager, Grundlagen, Eigenschaften und Anwendungen berührungsfreier, elektromagnetischer Lager.* Springer Verlag Berlin. ISBN 3-540-55868-3.

[Tolle, 1985] Tolle, H. (1985). *Mehrgrößen-Regelkreissynthese, Entwurf im Zustandsraum.* Oldenburg Verlag GmbH, München.

[Unbehauen and Nazaruddin, 1995] Unbehauen, H. and Nazaruddin, Y. (1995). Adaptive Zustandsregler für Mehrgrößensysteme und ihre praktische Anwendung der Zustandsraumdarstellung. *at-Automatisierungstechnik*, 43:236–241.

[Wurmsdobler, 1997] Wurmsdobler, P. (1997). *State Space Adaptive Control for a Rigid Rotor Suspended in Active Magnetic Bearings.* Dissertation, Institut für Maschinen- und Prozeßautomatisierung, Technische Universität Wien.