State Space Adaptive Control for a Rigid Rotor Suspended in Active Magnetic Bearings

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Abstract: *In this paper an adaptive state space controller for a rigid rotor suspended in active magnetic bearings is presented. For this sake a discrete time state space model is established in controller canonical form. Based on that model a recursive adaptation algorithm is used to estimate both all system parameters and all states. In addition a pole placement controller is calculated upon the identified model. Simulation results of the closed loop system including the proposed algorithm show the successful operation for changes of system parameters, in this case the sudden appearance of nonconservative cross-coupling forces, as generated by seals, for example.*

1 Introduction

In many active magnetic bearing applications the design of the entire control system is performed under the assumption that all system parameters are exactly known in advance and do not change during operation. For this type of plants adaptive control is not necessary. In high performance turbomachinery, however, a change in system parameters may easily occur, e.g. a sudden pressure loss may lead to a change in stiffness parameters of a sealing and finally to destabilising nonconservative cross-coupling forces, for example. In this case an adaptive controller is absolutely necessary. In the presented paper a rigid rotor is investigated with respect to the above described problem.

In section 2 the rotor model is shown. The rigid rotor is suspended by two active magnetic bearings with four degrees of freedom. Nonconservative cross-coupling forces are applied to the rotor in a given plane along the rotor axis. The continuous time state space model for the rotor bearing system is transformed into a discrete time innovations model with its system matrices in canonical form [4]. A certain prediction error algorithm which can be used to identify the state space model under on-line conditions is introduced in section 3 [1, 2]. The recursive algorithm, consisting of several matrix operations, can be performed very fast, which is important for a real-time implementation. Within this algorithm a state space model and all states are calculated after each sampling time interval. To provide numerical stability, a special implementation of this algorithm was used [5]. Section 4 considers the calculation of the state space controller by pole placement.

A change of the nonconservative cross-coupling parameters was simulated as well as the adaptation of the controller. The simulation results in section 5 show, that the proposed identification algorithm can cope with parameter changes and the entire system can be stabilised even for high values of the nonconservative stiffness coefficients.

2 State space rotor model

Figure 1 shows a sketch of the system under investigation. The rigid rotor is suspended by two active magnetic bearings at stations A' and B'. The proximity probes are assumed to be collocated with the bearings.

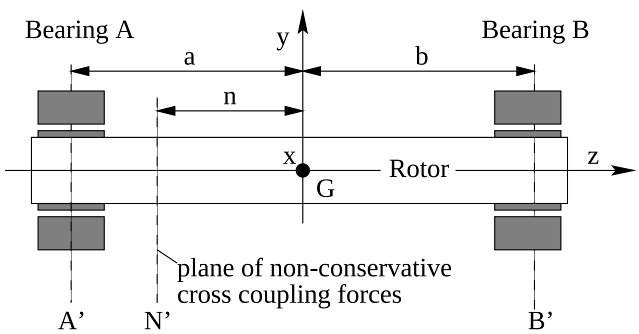

Fig. 1: Diagram of a rigid rotor suspended by two magnetic bearings and excited by non-conservative cross-coupling forces at Station N'. G denotes the centre of mass.

This is justified for a rigid rotor model, because the displacements in the bearing planes can easily be calculated from the sensor signal by simple transformation. The nonconservative forces are expected to act within the plane N'. All system parameters used for numerical simulation are shown in Table 1.

	Parameter	Value	Unit
m_r	rotor mass	28.7680	kg
I_a	axial mass moment of inertia	0.8632	kgm^2
I_p	polar mass moment of inertia	0.02188	kgm^2
a	distance to bearing A	0.23877	m
b	distance to bearing B	-0.24123	m
n	distance to plane N'	-0.1	m
k_s	AMB position stiffness	$1.758 \cdot 10^6$	N/m
k_i	AMB current stiffness	219.75	N/A
k	nonconservative cross-coupling stiffness	0 - 10^7	N/m
T_s	sampling time	10^{-4}	s

Tab. 1: System parameters used for a rigid rotor suspended by two magnetic bearings

2.1 Continuous time model

The second order differential equation for a rigid rotor with its degrees of freedom transformed into bearing coordinates $\mathbf{z} = [x_A, x_B, y_A, y_B]^T$ is given by

$$\mathcal{M}\,\ddot{\mathbf{z}} + \mathcal{G}\,\dot{\mathbf{z}} + (\mathcal{N} - \mathcal{K}_s)\,\mathbf{z} = \mathcal{K}_i\,\mathbf{u}, \tag{1}$$

with the input vector $\mathbf{u} = [i_{x_A}, i_{x_B}, i_{y_A}, i_{y_B}]^T$, respectively the control currents of four active magnetic bearing axes. All matrices are determined by a transformation from inertia coordinates $\mathbf{z}_c = \mathcal{T}\,\mathbf{z}$ in bearing coordinates $\mathbf{z}$ in the form

$$\begin{aligned} \mathcal{M} &= \mathcal{T}^T \mathcal{M}_s \mathcal{T}, & (2)\\ \mathcal{G} &= \mathcal{T}^T \mathcal{G}_s \mathcal{T}, & (3)\\ \mathcal{N} &= \mathcal{T}^T \mathcal{T}_n^T \mathcal{N}_n \mathcal{T}_n \mathcal{T}, & (4) \end{aligned}$$

with the mass matrix of the rotor

$$\mathcal{M}_s = \begin{pmatrix} I_a & 0 & 0 & 0 \\ 0 & m_r & 0 & 0 \\ 0 & 0 & I_a & 0 \\ 0 & 0 & 0 & m_r \end{pmatrix}, \tag{5}$$

its gyroscopic matrix

$$\mathcal{G}_s = \begin{pmatrix} 0 & 0 & I_p & 0 \\ 0 & 0 & 0 & 0 \\ -I_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Omega, \tag{6}$$

the geometrical transformation matrices

$$\begin{aligned} \mathcal{T} &= \begin{pmatrix} a & 1 & 0 & 0 \\ b & 1 & 0 & 0 \\ 0 & 0 & a & 1 \\ 0 & 0 & b & 1 \end{pmatrix}^{-1}, & (7)\\ \mathcal{T}_n &= \begin{pmatrix} n & 1 & 0 & 0 \\ 0 & 0 & n & 1 \end{pmatrix}, & (8) \end{aligned}$$

and the matrix of the nonconservative cross-coupling coefficients in the plane N'

$$\mathcal{N}_n = \begin{pmatrix} 0 & k \\ -k & 0 \end{pmatrix}. \tag{9}$$

The linearised model of the active magnetic bearings is determined by the matrices

$$\begin{aligned} \mathcal{K}_s &= k_s\,\mathbf{I}, & (10)\\ \mathcal{K}_i &= k_i\,\mathbf{I}. & (11) \end{aligned}$$

with $\mathbf{I}$ being the 4×4 identity matrix. The resulting continuous time state space model is then given in bearing coordinates in the form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathcal{A} \cdot \mathbf{x} + \mathcal{B} \cdot \mathbf{u}, & (12)\\ \mathbf{y} &= \mathcal{C} \cdot \mathbf{x}, & (13) \end{aligned}$$

with the state vector $\mathbf{x} = [\mathbf{z}, \dot{\mathbf{z}}]^T$ and the output vector $\mathbf{y} = \mathbf{z}$. The system matrices are defined as follows:

$$\begin{aligned} \mathcal{A} &= \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathcal{M}^{-1}(\mathcal{N} - \mathcal{K}_s) & -\mathcal{M}^{-1}\mathcal{G} \end{pmatrix}, & (14)\\ \mathcal{B} &= \begin{pmatrix} \mathbf{0} \\ \mathcal{M}^{-1}\mathcal{K}_i \end{pmatrix}, & (15)\\ \mathcal{C} &= \begin{pmatrix} \mathbf{I} & \mathbf{0} \end{pmatrix}. & (16) \end{aligned}$$

2.2 Discrete time model

A transformation into a discrete time system with sampling time T_s yields a model with no particular structure. Since the discrete state space model should have a minimum number of parameters to be identified, it is transformed into a canonical form. With respect to the calculation of an adaptive controller, a controller canonical form is chosen [3]. For the reason of symmetry the structural indices are $\nu = [2, 2, 2, 2]$, which means that there are four coupled second order systems. The entire model with system noise $\boldsymbol{\xi}(k)$ and measurement noise $\boldsymbol{\eta}(k)$ can then be written in the form

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,\mathbf{u}(k) + \boldsymbol{\xi}(k), & (17)\\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{x}(k) + \boldsymbol{\eta}(k). & (18) \end{aligned}$$

The state vector is $\mathbf{x}$, the input vector $\mathbf{u}(k)$ is the sampled input vector $\mathbf{u}$ and the output vector $\mathbf{y}(k)$ is the sampled vector $\mathbf{y}$. The system matrix $\mathbf{A} = \{\mathbf{A}^{(ij)}\}$ with $i, j = 1, 2, 3, 4$ and its sub-matrices

$$\mathbf{A}^{(ij)} = \begin{pmatrix} 0 & 1 \\ a_2^{(ij)} & a_1^{(ij)} \end{pmatrix}, \tag{19}$$

$$\mathbf{A}^{(ii)} = \begin{pmatrix} 0 & 0 \\ a_2^{(ii)} & a_1^{(ii)} \end{pmatrix}. \tag{20}$$

The control matrix $\mathbf{B} = \{\mathbf{b}^{(ij)}\}$ with

$$\mathbf{b}^{(ij)} = \begin{pmatrix} 0 & 1 \end{pmatrix}^T, \tag{21}$$

$$\mathbf{b}^{(ii)} = \begin{pmatrix} 0 & 0 \end{pmatrix}^T. \tag{22}$$

The measurement matrix is simply $\mathbf{C} = \{\mathbf{c}^{(ij)T}\}$ with

$$\mathbf{c}^{(ij)} = \begin{pmatrix} c_2^{(ij)} & c_1^{(ij)} \end{pmatrix}, \tag{23}$$

This model structure yields 64 parameters to be estimated, although all matrices have a total of 128 entries. Note, that these parameters are no longer parameters having a physical meaning, but the result of various transformations. Additionally the state vector is not identical with the sampled state vector of the originally continuous time model.

3 Estimation algorithm

The identification algorithm is based on the innovations model [4]:

$$\begin{aligned} \hat{\mathbf{x}}(k+1, \mathbf{p}) &= \mathbf{A}(\mathbf{p})\, \hat{\mathbf{x}}(k, \mathbf{p}) + \mathbf{B}(\mathbf{p})\, \mathbf{u}(k) + \\ &\quad \mathbf{K}(\mathbf{p})\, [\mathbf{y} - \hat{\mathbf{y}}(k|\mathbf{p})] \qquad (24) \\ \hat{\mathbf{y}}(k, \mathbf{p}) &= \mathbf{C}(\mathbf{p})\, \hat{\mathbf{x}}(k, \mathbf{p}) \qquad (25) \end{aligned}$$

with the Kalman matrix $\mathbf{K}(\mathbf{p})$. This matrix introduces 32 more parameters to the entire estimation algorithm, which means that there are 96 parameters to be estimated. All parameters of this innovations model are summarised within the parameter vector

$$\mathbf{p} = \left[a_1^{(11)}, a_2^{(11)}, \ldots\right]. \tag{26}$$

The goal of this estimation algorithm is to keep the estimation error $\mathbf{y} - \hat{\mathbf{y}}(k|\mathbf{p})$ as small as possible in terms of its statistical properties. Thus, it is necessary to minimise the error function

$$I(\mathbf{p}) = \frac{1}{2} \sum_{k=1}^{N} \varepsilon^2(k, \mathbf{p}) \rightarrow \text{Min.} \tag{27}$$

The minimisation of the error function results in the proposed estimation algorithm which can be summarised as [1, 2, 4]:

Prediction error

$$\varepsilon = \mathbf{y} - \hat{\mathbf{y}}(k|\mathbf{p}). \tag{28}$$

Adaptation of error matrix

$$\mathbf{L}(k) = \mathbf{P}(k-1)\, \mathbf{\Psi}(k) \left[\mathbf{I} + \mathbf{\Psi}^T(k)\, \mathbf{P}(k-1)\, \mathbf{\Psi}(k)\right]^{-1}. \tag{29}$$

Parameter update

$$\hat{\mathbf{p}}(k) = \hat{\mathbf{p}}(k-1) + \mathbf{L}(k)\, \varepsilon. \tag{30}$$

Update of covariance matrix

$$\mathbf{P}(k) = \frac{1}{\rho(k)} \left[\mathbf{P}(k-1) - \mathbf{L}(k)\, \mathbf{\Psi}^T(k) \mathbf{P}(k-1)\right]. \tag{31}$$

Based on the new model, a prediction for the next time step can be made in the form

$$\begin{aligned} \hat{\mathbf{x}}(k+1) &= \mathbf{A}(\mathbf{p}(k))\, \hat{\mathbf{x}}(k) + \mathbf{B}(\mathbf{p}(k))\, \mathbf{u}(k) + \\ &\quad \mathbf{K}(\mathbf{p}(k))\, \varepsilon, \qquad (32) \\ \hat{\mathbf{y}}(k+1) &= \mathbf{C}(\mathbf{p}(\mathbf{k}))\, \hat{\mathbf{x}}(k+1). \qquad (33) \end{aligned}$$

The gradient matrix $\mathbf{\Psi}$ is derived from the partial derivative of the estimation error as

$$\mathbf{\Psi}(k) = -\left(\frac{\partial}{\partial \mathbf{p}} \varepsilon(k)\right)^T = \left(\frac{\partial \hat{\mathbf{y}}(k, \mathbf{p})}{\partial \mathbf{p}}\right)^T, \tag{34}$$

and is calculated recursively by

$$\begin{aligned} \mathbf{W}(k+1, \hat{\mathbf{p}}) &= [\mathbf{A}(\hat{\mathbf{p}}(k)) - \mathbf{K}(\hat{\mathbf{p}}(k)) \mathbf{C}(\hat{\mathbf{p}}(k))] \mathbf{W}(k, \hat{\mathbf{p}}) \\ &\quad + \mathbf{M}_k - \mathbf{K}(\hat{\mathbf{p}}(k))\, \mathbf{V}_k, \qquad (35) \\ \mathbf{\Psi}(k+1) &= \mathbf{W}^T(k+1, \hat{\mathbf{p}})\, \mathbf{C}(\hat{\mathbf{p}}(k)) + \mathbf{V}_k^T. \qquad (36) \end{aligned}$$

The matrices $\mathbf{M}_k$ and $\mathbf{V}_k$ are model specific and result from the partial derivative of the innovations model as

$$\mathbf{M}_k = \frac{\partial}{\partial \mathbf{p}} [\mathbf{A}(\mathbf{p})\, \hat{\mathbf{x}}(k) + \mathbf{B}(\mathbf{p})\, \mathbf{u}(k) + \mathbf{K}(\mathbf{p})\, \varepsilon], \tag{37}$$

$$\mathbf{V}_k = \frac{\partial}{\partial \mathbf{p}} [\mathbf{C}(\mathbf{p})\, \hat{\mathbf{x}}(k)]. \tag{38}$$

Special attention has to be paid to the so-called forgetting factor $\rho(k)$ in equation 31. If this factor is too small ($\rho(k) \ll 1$), the covariance matrix will increase too fast, if it is too large (close to 1), the algorithm will react too slowly to parameter changes. Therefore, the forgetting factor needs to be controlled separately by a statistical value, which gives the change of the variance of the estimation error

$$\delta(k) = \frac{\hat{\sigma}_\varepsilon^2(N_1, k) - \hat{\sigma}_\varepsilon^2(N_2, k)}{\hat{\sigma}_\varepsilon^2(N_2, k)}, \tag{39}$$

with $N_1 \leq N_2$ and $\hat{\sigma}_\varepsilon^2(N, k)$ as estimate for the error variance with a variable memory N. The idea behind this algorithm is, that under the assumption of a stationary process, the variance of the estimation error in a steady state should be stationary as well. The increasing variance of the estimation error indicates a change in parameters of the plant under investigation. If $\delta(k)$ triggers a certain threshold, e.g. 20% of the mean variance, then the forgetting factor $\rho(k)$ is reset to ρ_0 and follows the law such that

$$\rho(k) = k_\rho \, \rho(k-1) + (1-k_\rho)\, \rho_\infty \tag{40}$$

with ρ_∞ the final value and k_ρ determining the rate of change.

Since one has to guarantee that the covariance matrix in equation 31 remains positive definite, a special algorithm for the covariance update has been used, based on the factorisation of the covariance matrix $\mathbf{P} = \mathbf{UDU}^T$ with $\mathbf{U}$ as upper triangular matrix and $\mathbf{D}$ a diagonal matrix [5]. These matrices are updated separately by this algorithm. Additionally, the diagonal matrix is increased by $r\,\mathbf{I}$ with $r > 0$, if the covariance becomes too small.

4 State Space controller

The control law for a state space controller is defined as

$$\mathbf{u} = -\mathbf{K}_x \, \mathbf{x}, \tag{41}$$

with $\mathbf{K}_x = \{\mathbf{k}_x^{(ij)T}\}$

The poles of the closed loop system are determined by the closed loop system matrix $\mathbf{A}_{cl} = \mathbf{A} - \mathbf{B}\mathbf{K}_x = \{\mathbf{A}_{cl}^{(ij)}\}$. with its sub-matrices:

$$\mathbf{A}_{cl}^{(ij)} = \begin{pmatrix} 0 & 1 \\ a_2^{(ij)} - k_2^{(ij)} & a_1^{(ij)} - k_2^{(ij)} \end{pmatrix} \tag{42}$$

$$\mathbf{A}_{cl}^{(ii)} = \begin{pmatrix} 0 & 0 \\ a_2^{(ii)} - k_2^{(ii)} & a_1^{(ii)} - k_2^{(ii)} \end{pmatrix} \tag{43}$$

The coefficients of the control matrix can be easily calculated in the form

$$k_m^{(ij)} = a_m^{(ij)}, \tag{44}$$

$$k_m^{(ii)} = a_m^{(ii)} + p_m^{(ii)}. \tag{45}$$

with $m = 1, 2$ and $i, j = 1, 2, 3, 4$. The coefficients $p_m^{(ii)}$ are derived from the desired polynomial for each subsystem

$$P_s^{(ii)}(z) = z^2 + p_1^{(ii)} z + p_2^{(ii)}.$$

Here, another advantage of a model description in controller canonical form becomes obvious. All parameters of the state space controller can be calculated directly from the state space model using simple algebra.

5 Simulation results

The simulation model presented in section 2 was simulated with the parameters as defined in Table 1. Further investigations have shown, that the nonconservative stiffness parameter k mainly affects the system matrix $\mathbf{A}$. Parameters of the measurement matrix $\mathbf{C}$ are influenced 10^{-4} times less than parameters of the system matrix. The same is true for the Kalman matrix $\mathbf{K}$. Therefore, these parameters need not to be estimated and only 32 entries remain in the parameter vector $\mathbf{p}$.

After the model has been defined with all parameters to be identified, the matrices $\mathbf{M}_k$ and $\mathbf{V}_k$ have to be calculated once according to equation (37). With these matrices the identification algorithm structure can be established, see equation (28) to (35).

Since the system is open-loop unstable, a controller and an observer have to be designed in advance. This has been done according to the nominal system without cross-coupling effects. The poles of the closed loop system have been placed at $z_{1,\ldots,8} = 0.97$, or in the continuous complex plane to $s_{1,\ldots,8} = -288\text{s}^{-1}$, which would meet the eigenvalue for a mass-spring-system with a negative stiffness factor of the magnetic bearing.

Similar to the controller, the observer poles have been chosen to $z_{1,\ldots,8} = 0.75$, which means that the observer is ten times faster than the closed loop system. Therefore, the Kalman matrix in equation 32 is assigned properly. The initial parameter vector is set to the a priori values of the nominal system, in this case all parameters of the system matrix $\mathbf{A}$ with the initial covariance matrix $\mathbf{P} = 1000\,\mathbf{I}$. All other recursively calculated matrices have been set to zero.

White noise with a maximum deflection value of $1\mu m$ for $\boldsymbol{\eta}$ is added to the measurement signal to ensure excitation. The simulation is started with initial conditions $\mathbf{x} = \mathbf{0}$ and all precalculated values for the controller and observer. The resulting displacements of the rotor at the bearing stations after a jump in the nonconservative stiffness coefficient can be seen in Figures 2 and 3.

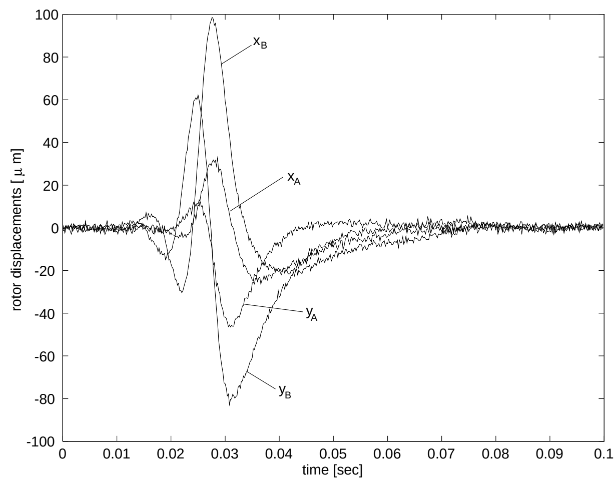

Fig. 2: Time history of the rotor displacements after a change in the nonconservative stiffness parameter k from 0 to 10^7 N/m at $t = 0.01$ s.

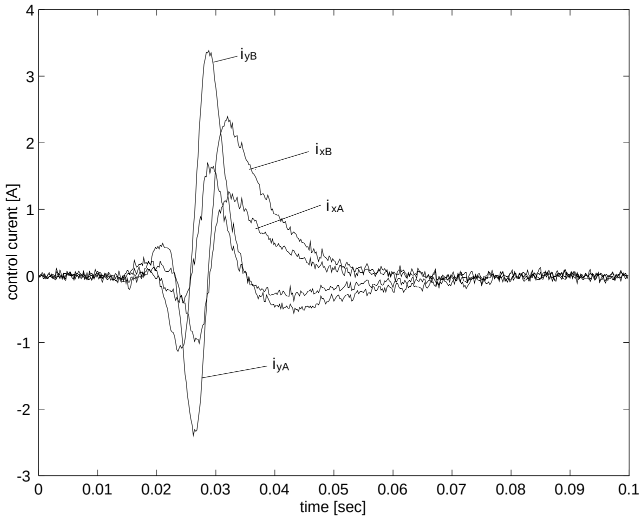

Fig. 3: Time history of the control currents after a change in the nonconservative stiffness parameter k from 0 to 10^7 N/m at $t = 0.01$ s.

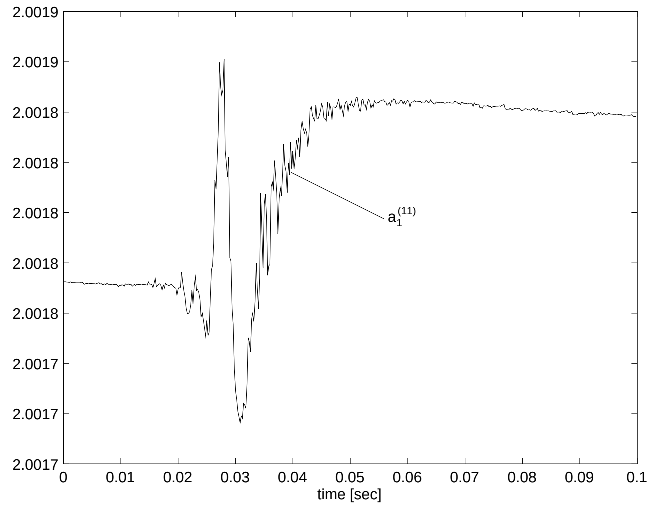

Fig. 5: Time history of a sample parameter a_1^{11} after a change in the nonconservative stiffness parameter k from 0 to 10^7 N/m at $t = 0.01$ s.

At $t = 0.01$ s the parameter k changes from 0 to $1 \cdot 10^7$ N/m. The jump is considered to be the worst case in turbomachinery application, e.g. a sudden pressure loss or leakage in sealings will lead to a parameter change in a linear model. Immediately after the parameter change the system is unstable in terms of its structure, but the displacements will increase slowly. When a certain signal to noise ratio is achieved, the parameter change is recognised by the algorithm and a change in the forgetting factor ρ is triggered (see Figure 4). This means, that the input and output signals have to contain a minimum on information to make the algorithm converge to the new system parameters (see Figure 5 and 6).

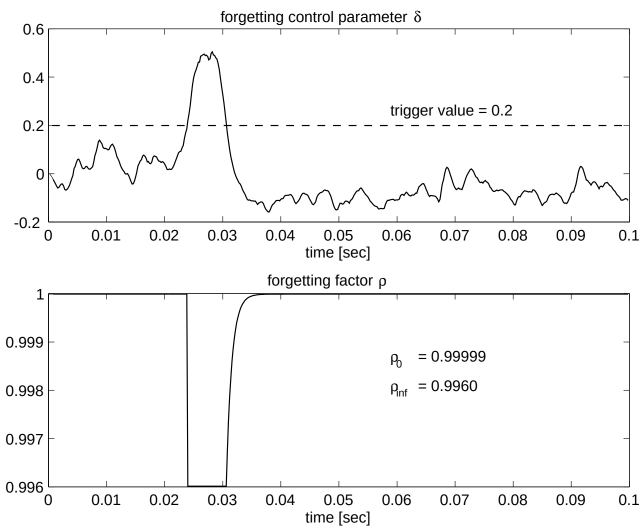

Fig. 4: Time history of forgetting control parameter δ and forgetting factor ρ after a change in the nonconservative stiffness parameter k from 0 to 10^7 N/m at $t = 0.01$ s.

Since 32 parameters have to be identified, it is not expedient to show the time histories of all parameters. Most of the estimated parameters of the system matrix do not change significantly, especially the parameters resulting from the rotor mass (Figure 5)., which appear at the diagonal of the system matrix, namely $a_2^{(ii)}, a_1^{(ii)}$. However, non-symmetric parameters are highly affected by the adaptation algorithm. A sample of the time history of those parameters can be seen in Figure 6.

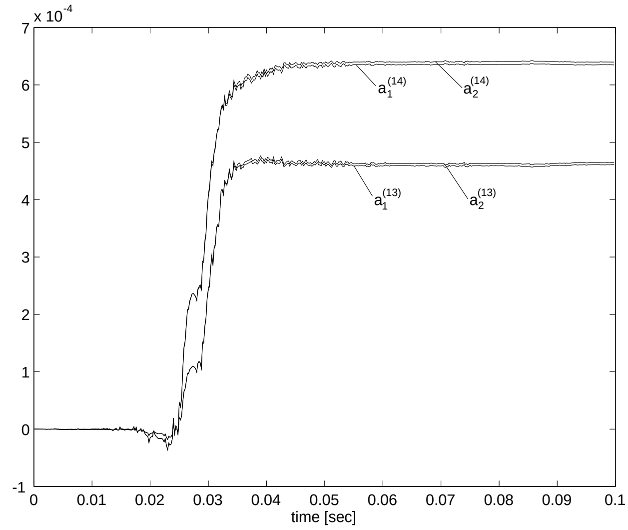

Fig. 6: Time history of sample parameters $a_2^{13}, a_1^{13}, a_2^{14}, a_1^{14}$ after a change in the nonconservative stiffness parameter k from 0 to 10^7 N/m at $t = 0.01$ s.

Simultaneously with the identification, the controller parameters are calculated according to equation (44) and (45). It can be observed, that the feedback gain matrix of the state space controller becomes non-symmetric to the same extent as the parameter of the nonconservative stiffness does. This effect can easily be seen in equation 44. These non-symmetric entries in the controller matrix will cause a force, which directly compensates the nonconservative forces of the system.

Due to the adaptation of the controller the rotor bearing system can recover from a parameter change. Simulations have shown, that this positive effect is true for a certain magnitude of the nonconservative forces. If the parameter change is too large (10^8 N/m), the control current becomes too large and the amplifier runs into saturation. Then, instability cannot be prevented.

6 Conclusion

It has been shown, that strongly time-invariant nonconservative cross-coupling forces in a rotor system can be controlled by means of on-line identification and adaptive control techniques. The system under investigation consists of a rigid rotor suspended by an active magnetic bearings controlled by a state space controller.

Within a numerical simulation a nonconservative cross-coupling force was applied to the system in conjunction with the proposed on-line identification and adaptive control. It turned out that a state space model identification is appropriate to estimate both the system parameters and the state vector. In this case, the nonconservative cross-coupling forces were compensated by additional forces generated by a non-symmetric gain in the state space controller matrix. This matrix is calculated by pole assignment from the identified state space model. This means, that the cross-coupling forces are compensated by counter cross-coupling forces.

Future objectives of investigation will be the extension of this concept by including integrative feedback and apply the algorithm to a higher sophisticated nonlinear simulation model. If the algorithm succeeds with this model, an implementation on a test rig will be carried out.

7 Acknowledgements

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