

# DIGITAL CURRENT CONTROL USING A SWITCHING POWER AMPLIFIER AND SIGMA DELTA MODULATION

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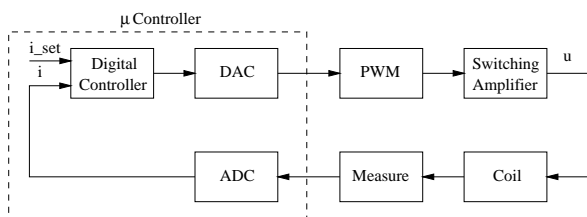
## Abstract

In this paper a new method for driving a switching power amplifier by use of sigma delta modulation is proposed. After an explanation of the principles of sigma delta modulation the digital control concept is presented. The conventional solution by means of both, digital to analogue conversion and pulse width modulation is replaced by a pure digital one. A digitally implemented sigma delta modulator directly drives a switching power amplifier. Experimental results show the success of this concept in controlling the current in an active magnetic bearing for the levitation of an iron ball.

## 1 Introduction

Switching power amplifiers are mostly used to supply inductive loads, e.g. the electromagnets in active magnetic bearings (AMBs). The current controller in this case is embedded in a position control, i.e. the magnetic suspension of an object, e.g. a rotor.

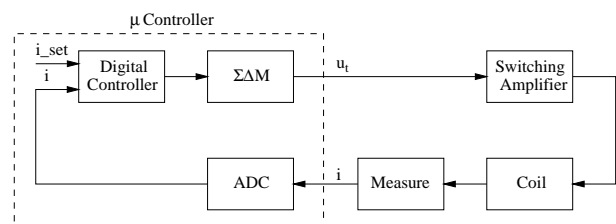
The conventional way the signals are processed in digital current control is depicted in Figure 1.



**Fig. 1:** Conventional solution of digital current control by means of pulse width modulator and switching power amplifier.

The input signal i.e. the measured current  $i$  is sampled by an analogue to digital converter (ADC) and then passed to the digital controller. After the computation of the control variable, the signal is converted back into the analogue domain by a digital to analogue converter (DAC). This signal is input to a pulse width modulator (PWM), which generates the control sequence for a switching amplifier.

If sigma delta modulation ( $\Sigma\Delta$ ) is used instead, both the DAC and the PWM are omitted and replaced by a tristate noise shaping converter in the digital domain [1]. The  $\Sigma\Delta$  directly computes the three state signal  $u_t$  encoded in  $1\frac{1}{2}$ -bit to control the switching amplifier. The third state (zero state) is introduced to prevent busy control action for signals of low magnitude [2]. In Figure 2 this new method is shown.



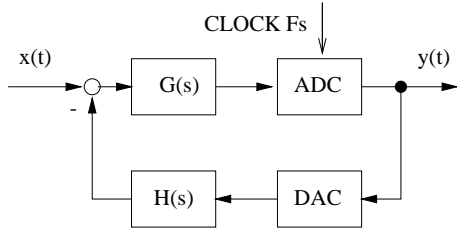
**Fig. 2:** Solution based on sigma delta modulation.

The result is a low-cost solution of a power electronics problem which uses a concept originating from communication techniques, namely employing a sigma delta modulator to drive a switching power amplifier. The implementation of this control concept is topic of this paper.

## 2 Principles of sigma delta modulation

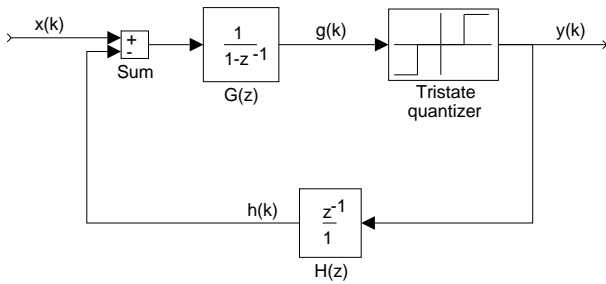
The basic idea behind sigma delta modulation is that the quantiser output is fed back to the input (see Figure 3)

[1, 4]. Thus, the quantisation error influences all following errors. This method is described as noise shaping conversion, i.e. the quantisation noise can be shaped by a filter  $H(s)$  in the feedback path. The transfer function  $G(s)$  in the forward path of Figure 3 in most cases consists of one or more integrators.



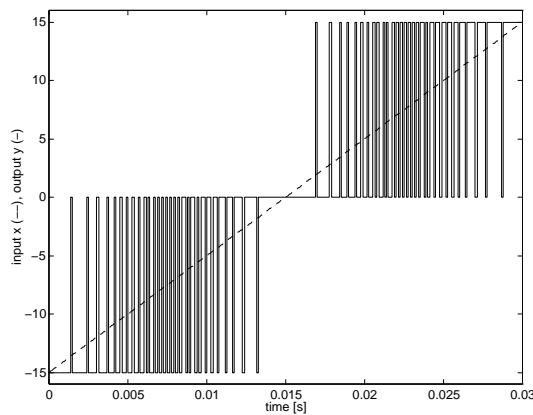
**Fig. 3:** The basic idea of sigma delta conversion.

The modulator in an analogue form can easily be transformed into a discrete time representation. A discrete time first order tristate noise shaping sigma delta modulator is shown in the MATLAB/Simulink block diagram in Figure 4.



**Fig. 4:** First order noise shaping sigma delta modulator, thresholds for non zero states are  $\pm 7.5$  [V] and output values  $\pm 15$  [V].

If a unit ramp is input to this converter, its operation becomes obvious. The resulting output can be seen in Figure 5.

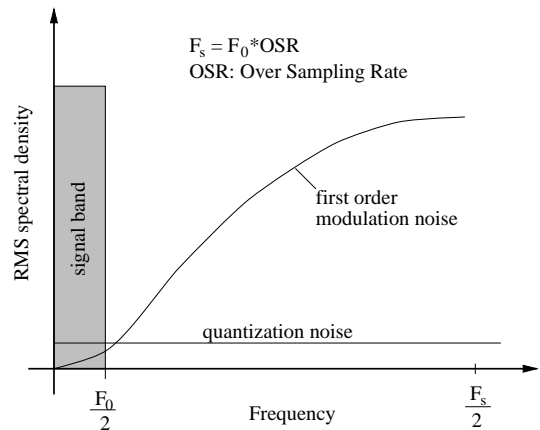


**Fig. 5:** The output of a first order tristate noise shaping sigma delta converter operating at 8kHz with the ramp  $1000\rho(t)$  as input.

The sigma delta modulator shown in Figure 4 represents a nonlinear discrete time system with feedback. If the input  $x(k)$  is persistently exciting with a constant spectral density over the interesting frequency band, the feedback loop in Figure 4 can be treated linearly. To achieve a linear model of the modulator in Figure 4 the quantiser is assumed to follow the equation  $y(k) = g(k) + e(k)$  where  $e(k)$  is the uniformly distributed quantisation error. This assumption leads to the equation for the entire modulation loop [1]

$$y(k) = x(k) + (1 - z^{-1}) e(k).$$

This means that the error term, which is the quantisation noise, undergoes a first order noise shaping with a low gain for low frequencies and a high gain for high frequencies. If a band limited white noise with the cut off frequency  $F_0/2$  is fed into the sigma delta modulator operating at  $F_m = F_0 \times \text{Over Sampling Rate (OSR)}$ , the output signal is corrupted by the modulation noise, which is a function depending on the noise shaping filter (Figure 6).



**Fig. 6:** Influence of sigma delta modulation on the output frequency spectrum.

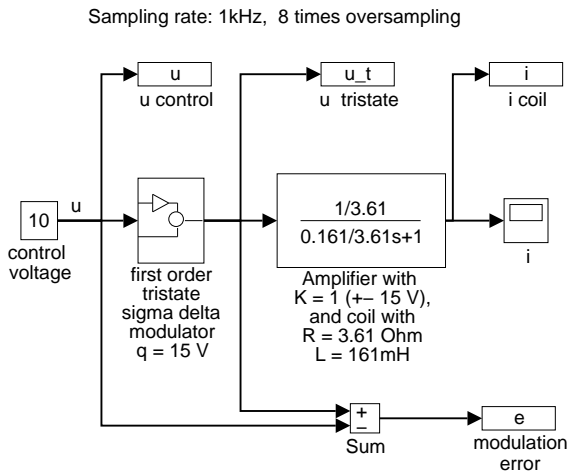
If the modulated signal is then input to a low pass with a cut off frequency of  $F_0/2$ , e.g. an inductive load, the modulation noise is suppressed and the original frequency band can be retrieved. This means that the sigma delta modulator can be viewed as unit gain within a certain frequency band. To keep the influence of the modulation noise low, a high oversampling rate must be provided. Additionally, more sophisticated noise shaping schemes will move the modulation noise outside the interesting frequency band.

### 3 Control by means of sigma delta modulation

In many digital control applications, a switching device is used to control the plant at the input, such as switching amplifiers for current control, or switching valves for pneumatic cylinders. For many applications pulse width modulation is used to derive a control sequence for these

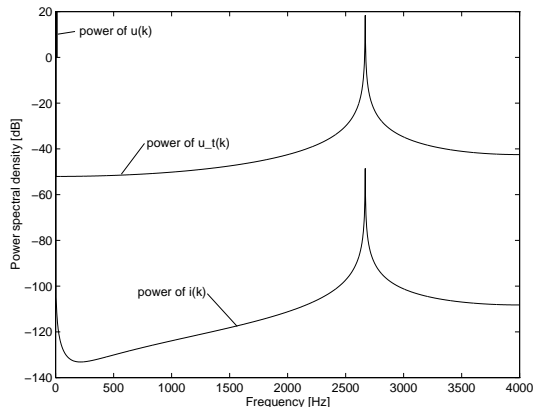
devices. In this section it is shown that sigma delta modulation is a good means to generate a control sequence for all applications of the described type.

The condition for the use of sigma delta modulation in digital control is a low pass behaviour of the plant. Then the modulation noise will not affect the frequency band of the controlled variable which guarantees the control variable to be independent of the modulation noise even in closed loop operation. Additionally, the modulator can only be treated linearly as long as its input is random and busy. However, in a steady state the control variable will be constant. Therefore, the following structure is investigated with respect to the described problem (Figure 7).



**Fig. 7:** Digital control using sigma delta modulation (open loop simulation).

A virtual output of a controller is assumed to be constant with  $u(k) = 10[V]$  and input to a sigma delta modulator operating at 8kHz. The  $1\frac{1}{2}$ -bit output of the modulator encoding three states is passed to the switching amplifier with  $U = \pm 15[V]$  which drives the coil. The coil has a resistance of  $3.61\Omega$  and an inductance of  $161mH$ , i.e. a cut off frequency of about  $22.24[1/s]$  or  $3.56[Hz]$ . The resulting power spectra of the tristate signal and of the coil current can be seen in Figure 8.



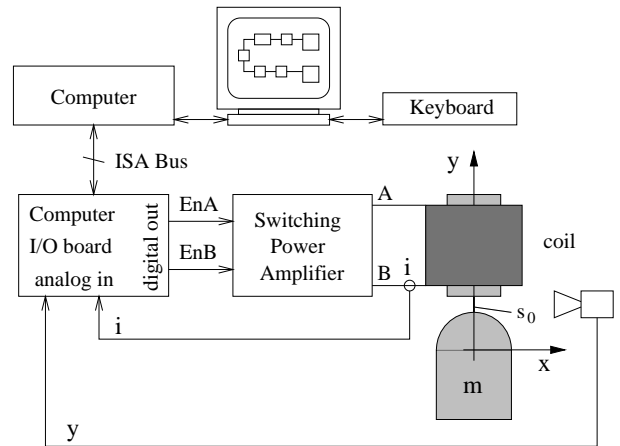
**Fig. 8:** Power spectra of sigma delta modulated tristate control signal  $u_t$  and of current  $i$  in the coil.

The spectrum of the control voltage  $u(k)$  is  $100\delta_0(\tau)$  with  $\delta_0(\tau)$  being the Kronecker Delta impulse. For the constant input to the modulator a periodic sequence is generated in the time domain with its frequency determined by  $U/u$ . The frequency spectrum renders the Fourier coefficients for the periodic signal added to the stationary value of  $100\delta_0(\tau)$ .

As one can see, the low pass behaviour of the coil filters out high frequency parts to a certain extent, in this case -50dB. Since the interesting frequency band is limited by 1kHz with the controller running at 8kHz, the higher frequencies can be cut off additionally by a digital filter. Therefore, sigma delta modulation is applicable for the control of plants with switching actuators and low pass behaviour, because it can be regarded as unit gain.

## 4 Control applications

This new approach to digital current control was tested on a magnetically levitated iron ball as a simplified model of an active magnetic bearing. The low cost test setup can be seen in Figure 9 with all physical parameters listed in Table 1.



**Fig. 9:** Experimental setup for the implementation of the control concept based upon sigma delta modulation.

Parameter	Value	Unit	
$m$	0.16	[kg]	mass of iron ball
$s_0$	17	[mm]	nominal air gap
$r$	3.61	[ $\Omega$ ]	ohmic resistance of coil
$L$	161	[mH]	inductance of coil

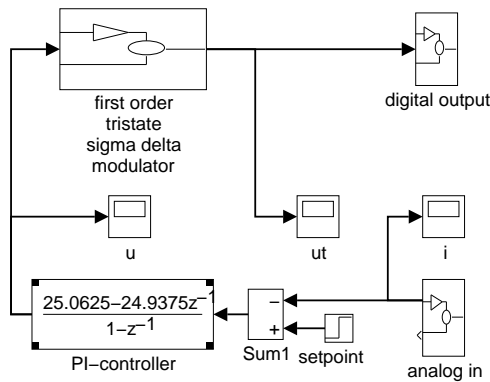
**Tab. 1:** Most important parameters of the test setup.

The computer used as controller is equipped with an I/O board with two digital outputs to drive the switching amplifier, and analogue inputs for the measurement of the control current and the position of the levitated iron ball. The switching amplifier consists of a full bridge in an integrated circuit and a power supply. The tristate signal is

encoded in 2 digital lines. The supply voltage of +15[V] is switched to port A, if EnA is high and to port B, if EnB is high. If both EnA and EnB are low (zero state), port A and B are short circuited via fast recovery diodes.

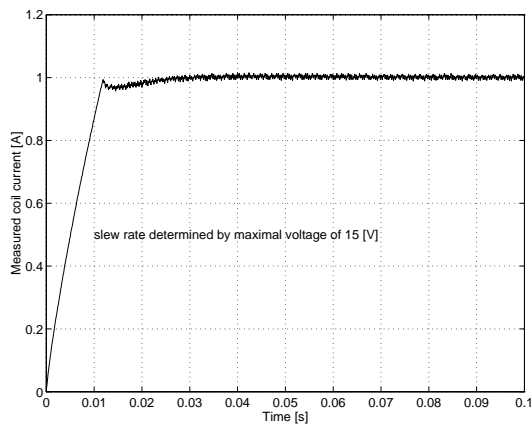
#### 4.1 Digital current control

The system described by Figure 9 with the parameters of Table 1 was tested in closed loop operation. The current in the coil was controlled by a digitally implemented PI-controller operating at 8kHz with a proportional gain  $K_P = 25$  and an integral gain  $K_I = 1000$ . The MATLAB/Simulink block diagram used for compilation to a real time application by means of the MATLAB/Simulink real time workshop (RTW) [6] is depicted in Figure 10.

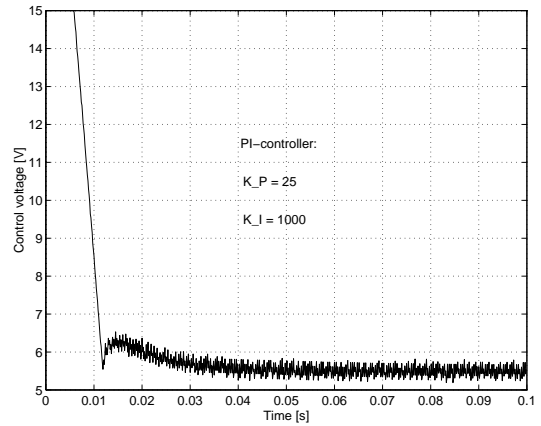


**Fig. 10:** Block diagram of the current control loop.

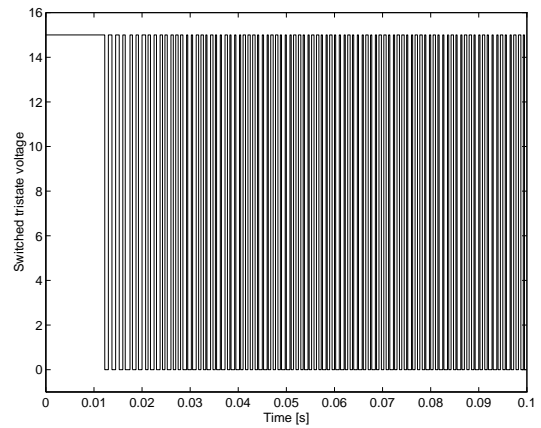
The closed loop performance was tested by a step response with 1[A] as set point. The current transient can be seen in Figure 11, the control variable  $u$  in Figure 12 and the digital tristate signal in Figure 13.



**Fig. 11:** Step response of coil current  $i$ .



**Fig. 12:** Control variable  $u$  for a step response.



**Fig. 13:** Digital tristate signal  $u_t$  for a step response.

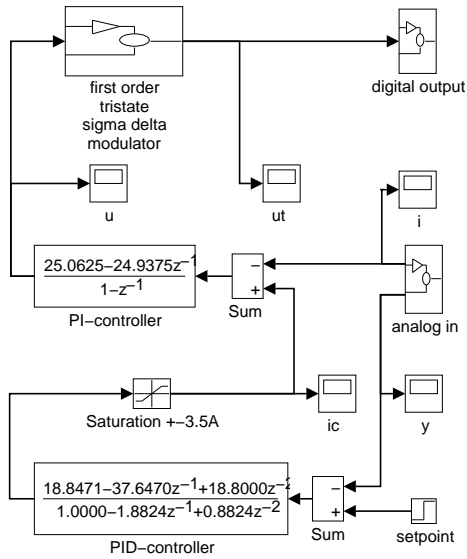
In the present case only two states of the switching amplifier are used, namely +15[V] and 0[V]. Heuristically speaking, the controller injects a voltage impulse, i.e. energy, into the system, if it is required, even if the control error is small.

Since the resistance is rather high and the inductance small, the theoretical requirements mentioned in the previous section are only fulfilled to a certain extent, i.e. the cut off frequency is too large. Therefore, the current signal is corrupted by a relatively large ripple. The amplitude of the ripple depends on the ratio between the switching frequency and the cut off frequency of the inductive load which determines the over sampling rate.

The noise spectrum of the ripple mainly depends on the stationary value of the control voltage as shown in the previous section (power spectral density of  $i(k)$ ). The main frequency of this ripple renders the periodicity of the modulator at a constant input. This effect could be deteriorated by using a higher OSR. Then the disturbing frequencies are shifted to higher harmonics with smaller residuals.

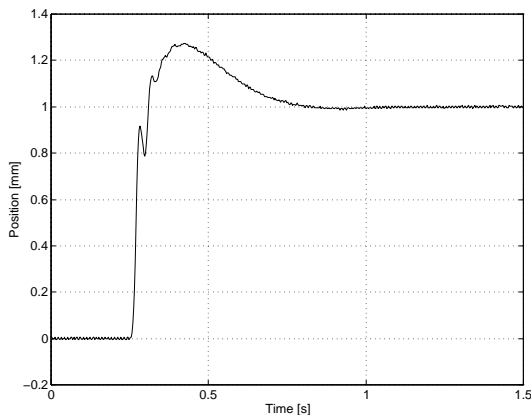
## 4.2 Digital position control

Since current control by means of sigma delta modulation works properly, the current control loop is embedded in a position control loop. A digitally implemented PID-controller operating at 8kHz has the proportional gain  $K_P = 0.4$ , the derivative gain  $K_D = 0.02$  and the integral gain  $K_I = 2$ . This controller was implemented digitally as well. The MATLAB/Simulink block diagram used for compilation to a real time application can be seen in Figure 14.

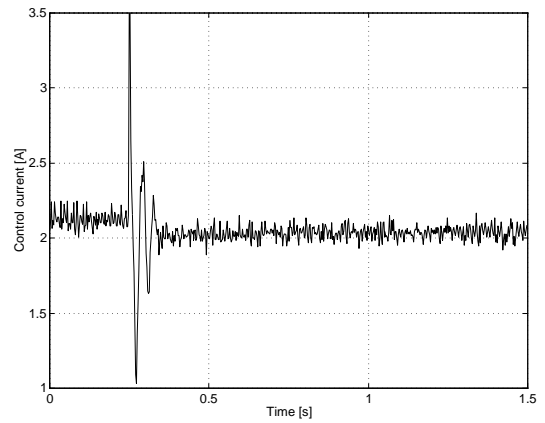


**Fig. 14:** Block diagram of the entire control loop for the levitation of an iron ball.

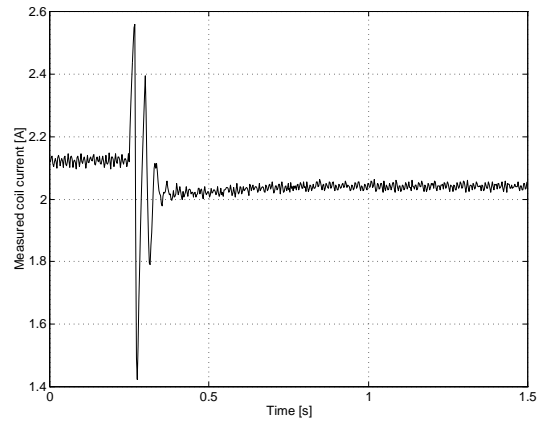
The closed loop operation of the position control loop with underlying current control loop was tested by a step response to a set point change of the ball position. A position step response can be seen in Figure 15, the corresponding control current in Figure 16, the measured current in Figure 17, the control voltage in Figure 18 and the digital tristate signal in Figure 19.



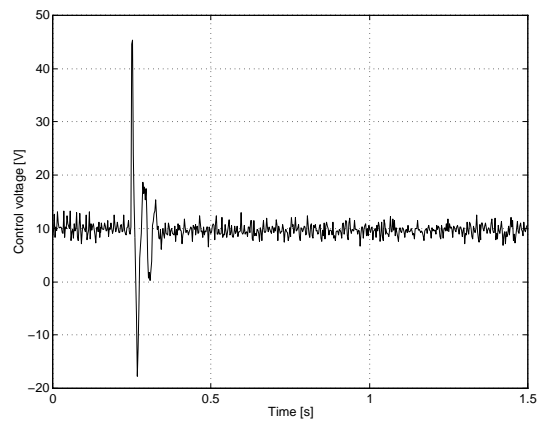
**Fig. 15:** Position  $y$  for a step response.



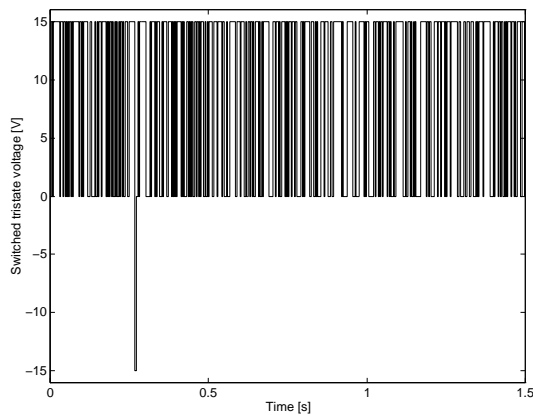
**Fig. 16:** Control current  $i_c$  for a step response.



**Fig. 17:** Measured current  $i$  for a step response.



**Fig. 18:** Control voltage  $u$  for a step response.



**Fig. 19:** Digital tristate signal  $u_t$  for a step response.

As the experimental results show, the control concept using sigma delta modulation basically works, although the OSR is rather small. The performance could be improved by a larger OSR, e.g. 32. This will result in a lower ripple both on the measured current and on the measured position of the levitated object.

In a real active magnetic bearing application all time constants are smaller. In so far, the experimental results presented in this paper have to be regarded as basic results. The concept can easily be adapted to a real active magnetic bearing application, if the required hardware is available, e.g. the implementation of the sigma delta modulator operating at 200kHz within a digital signal processor (DSP) in conjunction with a switching power amplifier.

## 5 Conclusion

A new concept to drive a switching power amplifier in order to control a current in a coil was presented and applied to the control of magnetically levitated object as simplified model for an active magnetic bearing. This method makes use of sigma delta modulation instead of conventional digital to analogue conversion in conjunction with pulse width modulation. The entire control concept was implemented on a digital computer.

The advantages of this concept are obvious. Two integrated circuits, namely a DAC and a PWM can be omitted, since the modulation can be carried out within a digital controller. In addition, the signal path from the digital controller to the switching amplifier can be implemented as a digital connection, e.g. by glass fibre.

On the other hand, a sigma delta modulator introduces a more wide spread noise into the analogue system. In comparison, the noise spectrum of a pulse width modulator is dominated by the switching frequency with all higher harmonics with its amplitudes depending on the magnitude of the control signal. For a sigma delta modulator, even the frequencies depend on the amplitude. If

the control variable changes its magnitude during operation, a wide spectrum results.

Of course, a PWM could be implemented digitally as well. But for this purpose an additional timer is needed to simulate a PWM. Since this can only be done at discrete instants, jitter is generated. A  $\Sigma\Delta$  operates at a certain frequency *a priori*. Since the  $\Sigma\Delta$  will operate at higher frequencies than a PWM, the control concept is subjected more to jitter due to hardware effects. However, as hardware becomes more reliable, this problem is extinguished.

A weak point of the control concept is that the clock frequency has to be higher with a  $\Sigma\Delta$  in comparison to a PWM. However, a  $\Sigma\Delta$  guarantees a minimum switch-on time with longer switch-on times possible during operation. Additionally, the tristate modulator is in an idle mode for long periods for small signal magnitudes. For a PWM it is the other way round.

A future objective will be to additionally replace the analogue to digital converter by a sigma delta modulator as simple hardware device producing a 1-bit signal. All signal processing can be done by one controller with only digital connections to simple analogue devices, such as a  $\Sigma\Delta$  at the input and a switching amplifier at the output. If the digital controller should run at a lower frequency, the  $\Sigma\Delta$ s at the input and output can be implemented as ASICs.

Moreover, the concept can be applied to all plants with switching control devices, like switching valves in the position control of pneumatic cylinders. The goal of this control concept is to shift devices from the analogue domain to the digital domain in order to gain more reliability.

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